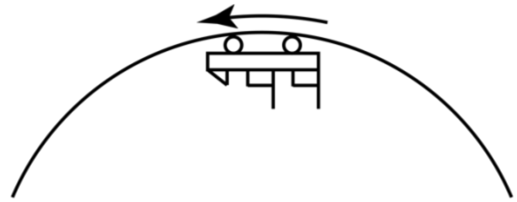
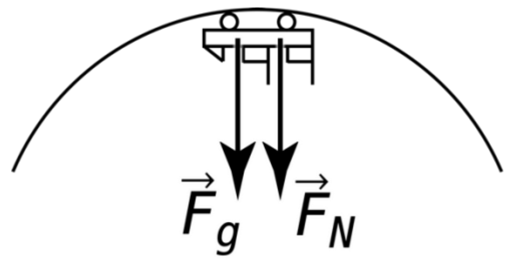
1) A rollercoaster car moves through a vertical loop as shown in the figure. The relationship between the magnitude of the force normal, F_N, acting on the car, and the magnitude of the force of gravity, F_g, acting on the car must be which of the following.

(A) $F_N > F_g$ (B) $F_N = F_g$ (C) $F_N < F_g$
(D) The magnitudes cannot be compared without knowing the speed of the car.



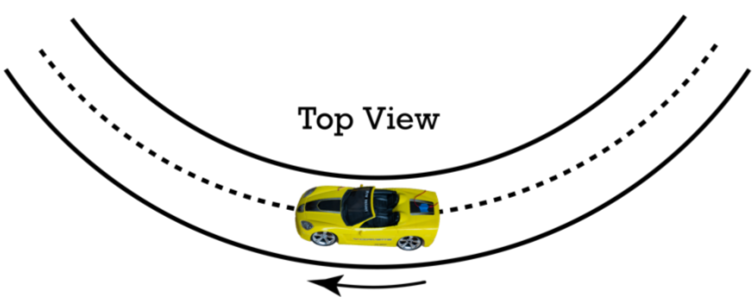
In order to determine the relationship between the magnitudes of the force normal and force of gravity, we need to sum the forces in the in-direction. Both the force normal and force of gravity act down or in towards the center of the circle. The force normal and force of gravity are both positive because they act inward, toward the center of the circle.

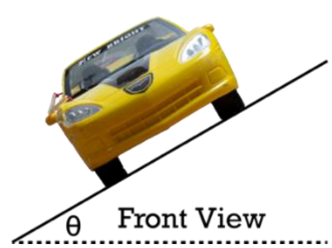


$$\sum F_{in} = F_N + F_g = ma_c = m\left(\frac{v_t^2}{r}\right) \Rightarrow F_N = \frac{mv_t^2}{r} - F_g$$

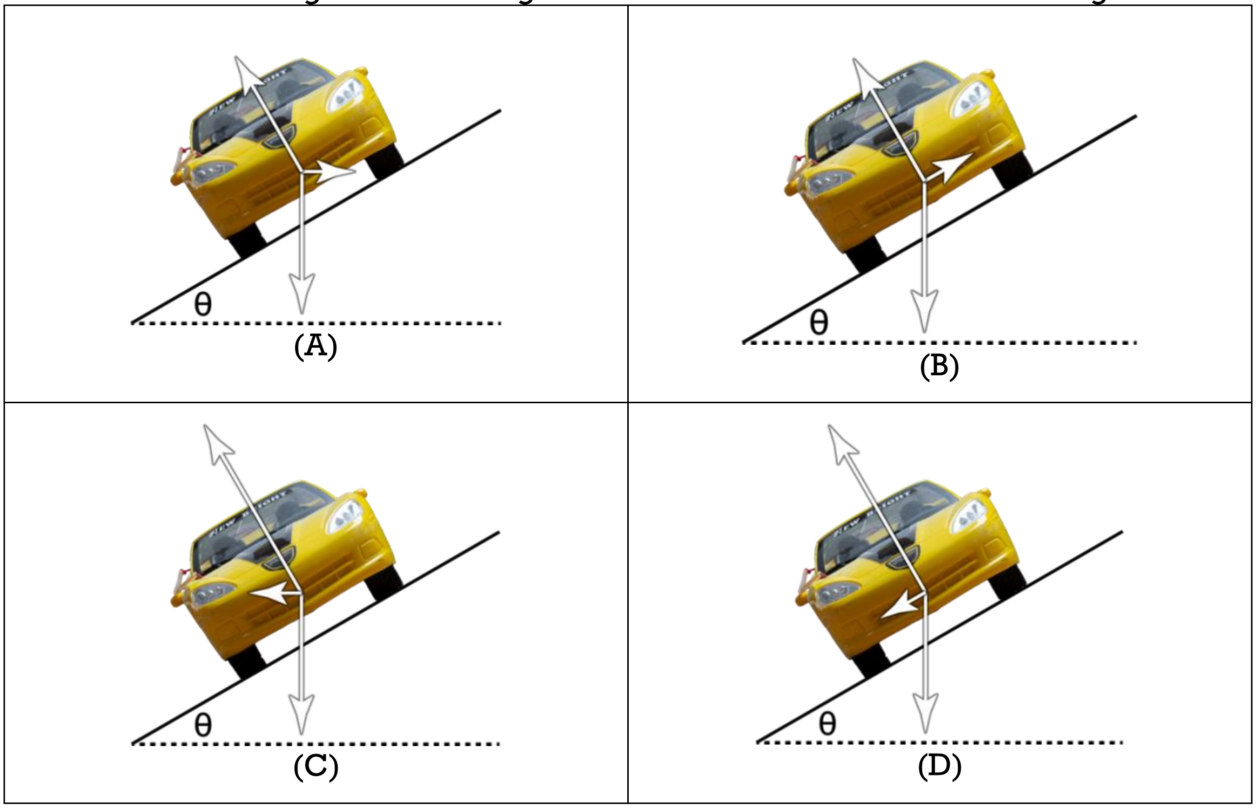
In other words, the magnitude of the force normal is dependent on the tangential speed of the car. A larger tangential speed of the car will result in a larger force normal, and a smaller tangential speed of the car will result in a smaller force normal. The magnitude of the force normal could be more or less than the magnitude of the force of gravity. It just depends on the tangential speed of the car. The correct answer is D.

2) As shown in the top and front view illustrations, a car moves along a banked curve. In other words, the car is moving in a circle on an incline. The car is not sliding relative to the incline.

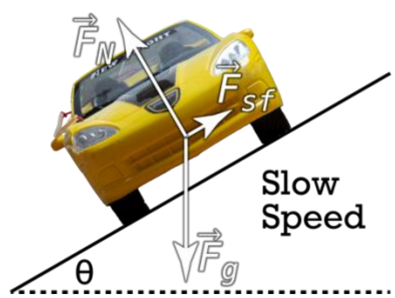


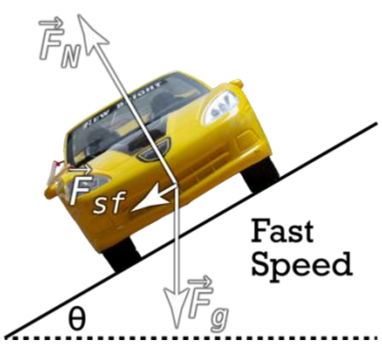


Which of the following **two** force diagrams could illustrate all the forces acting on the car.



- *The force of gravity acts straight down. This fits all answer choices.*
- *The force normal acts perpendicular to the incline and is a push, therefore, the force normal is up and perpendicular to the incline. This fits all answer choices.*
- *The force of static friction acts parallel to the incline and opposes the sliding motion of the car tires relative to the incline.*
  - *If the car is moving slowly enough, the force of static friction prevents the car from sliding down the incline, therefore, the force of static friction acts up the incline. This is choice B.*
  - *If the car is moving quickly enough, the force of static friction prevents the car from sliding up the incline, therefore, the force of static friction acts down the incline. This is choice D.*





The correct answers are choices B and D.

A couple of interesting things to point out about the two above free body diagrams. First off, you might be wondering why the force normal for the car which is moving more slowly has a smaller force normal. Well, realize that, because the car is not accelerating *in the y-direction, the net force in the y-direction has to equal zero. (This assumes the car is moving at a constant speed*♥*, which is a reasonable assumption.)*

When the car is moving at a ***slow*** *speed, we get the following:*

$$\text{slow: } \sum F_y = F_{N_y} + F_{sf_y} - F_g = ma_y = m(0) = 0 \Rightarrow F_{N_y} = F_g - F_{sf_y}$$

Therefore, the y-component of the force normal when the car is moving at a ***slow*** *speed equals the force of gravity* ***minus*** *the y-component of the force of static friction acting on the car.*

When the car is moving at a ***fast*** *speed, we get the following:*

$$\text{fast: } \sum F_y = F_{N_y} - F_{sf_y} - F_g = ma_y = m(0) = 0 \Rightarrow F_{N_y} = F_g + F_{sf_y}$$

The y-component of the force normal when the car is moving at a ***fast*** *speed equals the force of gravity* ***plus*** *the y-component of the force of static friction acting on the car.*

Therefore, because the force normal is in the same direction in both instances, and because the y-component of the force normal is smaller when the car is moving more slowly, the force normal is smaller when the car is moving more slowly.

Also, the net forces in the in-direction when the car is moving slowly and quickly are:

$$\text{slow: } \sum F_{in} = F_{N_{in}} - F_{sf_{in}} = ma_c = m\left(\frac{v_t^2}{r}\right) \text{ \& fast: } \sum F_{in} = F_{N_{in}} + F_{sf_{in}} = m\left(\frac{v_t^2}{r}\right)$$

In other words, the net force in the in-direction, or the centripetal force acting on the car, when the car is moving more quickly is larger, this is because the horizontal component of the force of static friction acts inward when the car is moving faster and because the tangential speed of the car is larger.

Lastly, you may be asking yourself, "If the force of static friction equals the coefficient of static friction times force normal, how can the force of static friction in the two instances have roughly the same magnitude? When the car is moving faster, the force normal is larger, so the force of static friction should be larger, right?"

To which I reply, "Does the force of static friction really equal the coefficient of static friction times force normal?"

To which you should definitely begrudgingly reply, "No, darn it, I made that same mistake again that I made so many times in the 'Friction - AP Physics 1: Dynamics Review Supplement'. The ***maximum*** *force of static friction equals the coefficient of static friction times force normal."*

And I reply, "Yes, please, please, please remember the less than or equal to sign in the force of static friction equation!" $F_{sf} \le \mu_s F_N$

♥ *Not constant velocity because the direction of the velocity of the car is changing.*